Physics Olympiad «Phystech», February 2023

Problem Set 10-03

Ordinary fractions and radicals are allowed in your answers in all the problems of the set.

1. A projectile flies vertically and explodes at the highest point of the trajectory into many fragments flying in all possible directions with equal velocities in modulus. In $t_1 = 0.4$ s after the explosion all the fragments are in flight, one of the fragments moves horizontally, its momentum is $P_1 = 30$ kg·m/s. The mass of the projectile is $M = 10$ kg.

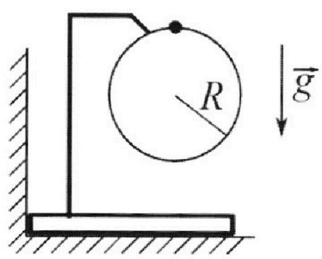
1) Find the module P_2 of the total momentum $\vec{P}_2$ of all the other fragments at this time point. Acceleration due to gravity is $g = 10$ m/s^2.

2) Find the angle α between the vectors $\vec{P}_2$ and $\vec{g}$ at this time point. In the answer specify the value of the trigonometric function of the angle α: sin α or tan α.

The maximum distance from the point of explosion to the ground point of impact equals $d = 80$ m.

3) Find the duration T of the flight of such fragments. Air resistance can be ignored.

2. The bar is installed close to the vertical wall (see Fig.). The ring of radius $R = 1$m is fixed on the bar (the ring is in the vertical plane). A ball is put on the ring. The masses of the bar and the ball are the same. The ring and the holder are lightweight. There is no friction. From the upper point of the ring the ball slides with a negligible initial velocity.



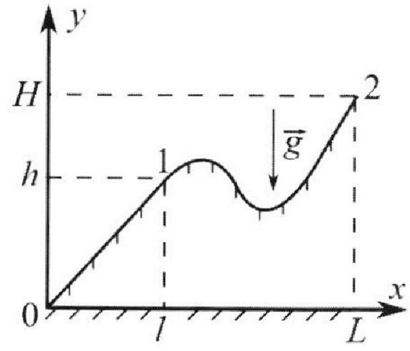
1) Find the acceleration $\vec{a}$ of the ball at the moment when the force with which the bar acts on the vertical wall turns to zero. In the answer specify the module and direction of the vector $\vec{a}$.

2) Find the vertical displacement h of the ball to this point of time.

3) Find the maximum speed V of the bar movement.

Acceleration due to gravity is $g = 10$ m/s^2. In the process of movement the bar does not break away from the smooth horizontal surface.

3. A schoolboy pulls a sled on a hill moving in the straight line. Mass of the sled is $m = 5$ kg. The profile of the hill in the vertical plane is shown in the drawing for the task. In order to pull the sled slowly from point 0 to point 1, applying force along the flat surface of the hill, it is necessary to perform work $A_1 = 300$ J. At point 1 the_schoolboy releases the sled. The vertical coordinate of the starting point is $h = 4.6$ m, the initial velocity of the sled is zero. The kinetic friction coefficient is the same on the entire surface of the hill. Acceleration due to gravity is $g = 10$ m/s^2.



1) Find the speed V of the sled at the base of the hill at point 0.

2) Find the work A_2 that should be done to move the sled slowly from point 1 to point 2? The vertical coordinate of the point 2 is $H = 10$ m, $L = 4l$. At each elementary displacement the vector of the force that the schoolboy applies to the sled and the vector of the sled displacement lie on the same straight line. All displacement occur in the same vertical plane.

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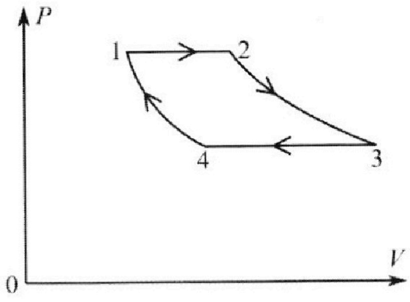
Ordinary fractions and radicals are allowed in your answers in all the problems of the set.

4. In the 1-2-3-4-1 cycle of the heat engine, there are two isobars and two isotherms (see Fig.). The working substance is a monatomic ideal gas. In the process of isobaric expansion up to double the volume, the gas performs the work A. The gas performs the same work A during isothermal expansion.

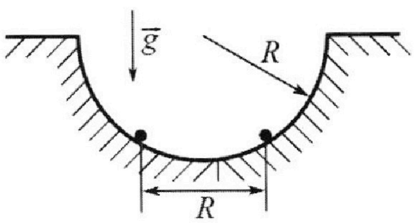


1) Find the amount Q of heat added to the gas in processes 1-2-3.

2) Find the amount Q_{34} of heat removed from the gas in the process of isobaric compression $(Q_{34} > 0)$.

3) Find the efficiency e of the cycle.

5. A hemispherical hole of radius R is made in a smooth horizontal surface. Two charged balls are held in the hole at the same horizontal level (see Fig.). The mass of each ball is m, the distance between the balls is R. The balls are simultaneously released and they come off the hole at the edges. There is no friction. Counted from the edge of the hole the maximum height to which each ball rises in flight is equal to R.



1) Find the speed V of each ball in the beginning of the flight.

2) Find the charge Q of each ball.

3) Find the maximum speed U with which the distance between the balls grows after beginning the flight.

Acceleration due to gravity is g. The collisions of the balls with the horizontal surface are absolutely elastic. The proportionality coefficient in Coulomb's law is k.

You can present **only one** task on one page.
Please put a cross against a number of the task
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MIPT

1. At the highest point of trajectory (page 1)
the velocity is 0

$P_0 = m v$ $v = 0$

$P_0 = 0$

$$\Delta P = \Sigma F \cdot \Delta t$$

$$P_3 - P_0 = \Sigma_F \cdot \Delta t$$

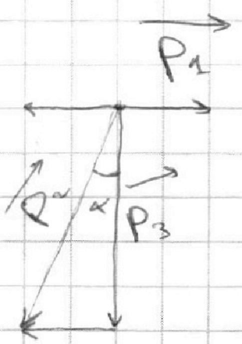
$$P_3 = \sum_{i=0} (m_i \cdot g) \cdot \Delta t$$

$$P_3 = M \cdot g \times \Delta t$$

$$P_3 = 10 \times 10 \times 0.4 = 40 \text{ kgm/s}$$

P_3 is the net momentum of all fragments

$$\vec{P_2} = \vec{P_3} + \vec{P_1}$$



$$P_2 = \sqrt{(30)^2 + (40)^2}$$

$$P^2 = \sqrt{900 + 1600}$$

$$P = 50 \text{ kgm/s}$$ Answer

2) $\tan\alpha = \frac{30}{40} = \frac{3}{4}$

$\tan\alpha = \frac{3}{4}$: Answer

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MIPT

page (2)

$\Delta P = \sum F_i . \Delta t$

$P_i = m_i g \Delta t$

$~~m_i~~ \mu = ~~m_i~~ g \Delta t$

$\mu = 0.4 \times 10$

$\mu = 4 \ m/s$

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MIPT

2] $dW = -m_1 g \sin\Theta\, ds - \mu m_1 g \cos\Theta\, ds + f_1 ds$

$dw = [F - m_1 g(\mu \cos\Theta + \sin\Theta)]ds$

$W = m_1 g [H - h] + \mu m_1 g \cos\Theta [S_{12}]$]

$w = m_1 g \left[[H-h] + \mu \cos\Theta [S_{12}] \right]$

~~Where S is the~~

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MIPT

1] From point 0 to 1

$\Delta E_k + \Delta E_p + U = 300$ where U is energy lost to friction.

$0 + m.g(\Delta h) + U = 300$

$U = -m.g\,h + 300$ $\quad \mu = m.g\,\mu_k \cos\theta$

~~$m.g\,\mu_k \cos\theta = m.g\,h + 300$~~ ~~cos~~

$U = -5 \times 10 \times 4.6 + 300$

$U = -5 \times 46 + 300$

$U = 70$ J

From point 1 to 0

$\Delta E_k + \Delta E_p + U = 0$

$\frac{1}{2} m v^2 + (-230) + 70 = 0$

$\frac{1}{2} m v^2 = 160$

$v = \sqrt{\frac{2 \times 160}{5}} = \sqrt{\frac{2 \times 2 \times 5 \times 16}{5}} = \sqrt{4 \times 16} = 2 \times 4 =$

Answer $\boxed{v = 8\ m.s^{-1}}$

2]

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MIPT

1) in the process 1-2

$$\Delta U = W + Q_{12}$$

$W = \int_{V_1}^{V_2} P\,dV$ $\quad P = const$ (isobaric process)

$W = P_1(V_2 - V_1) = P_1(2V_1 - V_1) = P_1 V_1$ $\quad A = W = P_1 V_1$

$\Delta U = n C_V \Delta T$ $\quad P_1 V_1 = nRT_1$

$P_2 V_2 = nRT_2$

$n C_V (T_2 - T_1) = W + Q_{12}$

$P_1 V_1 = nRT_1$

$n C_V (T_1) = P_1 V_1 + Q_{12}$ $\quad P_1 (2V_1) = nRT_2$

$n C_V (T_1) - P_1 V_1 = Q_{12}$ $\quad \frac{1}{2} = \frac{T_1}{T_2}$ $\quad T_2 = 2T_1$

in the process 2-3

$\Delta U = W + Q_{2-3}$ $\quad \Delta U = 0$ (isothermal process)

$T = const$

$0 = -A + Q_{2-3}$

$Q_{2-3} = +A$ $\quad$ which means heat left the system

$Q_{2-3} = +P_1 V_1$

~~$Q_{1-2-3} = W_{1-2} + W_{2-3}$~~ $\quad Q_{1-2-3} = Q_{1-2} + Q_{2-3}$

$Q_{1-2-3} = C_V \left(\frac{2 P_1 V_1}{R}\right) - \cancel{P_1 V_1} + \cancel{P_1 V_1}$

$Q_{1-2-3} = P_1 V_1 \left(2\frac{C_V}{R}\right)$ Answer $Q_{123} = P_1 V_1 \frac{2C_V}{R}$

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MIPT

2

$$+A = \int_{V_2}^{V_3} \frac{2nRT_1}{V} dV$$

$$+P_1V_1 = 2nRT_1 [\ln V]_{2V_1}^{V_3}$$

$$+P_1V_1 = 2nRT_1 \ \ln V_3 - \ln 2V_1$$

$$\frac{+P_1V_1}{2nRT_1} = \ln V_3 - \ln 2V_1$$

$$\exp\left(\frac{+P_1V_1}{2nRT_1}\right) = \frac{V_3}{2V_1}$$

$$2\exp\left(\frac{+P_1V_1}{2nRT_1}\right)V_1 = V_3$$

$$2\exp\left(\frac{P_1V_1}{2nR\left(\frac{P_1V_1}{nR}\right)}\right)V_1 = V_3$$

$$2\exp\left(\frac{1}{2}\right)V_1 = V_3$$

$$P_3 = \frac{2nRT_1}{V_3}$$

$$V_4 = \frac{nRT_1}{P_3}$$

$$P_3 = P_1\exp\left(\frac{-1}{2}\right)$$

$$V_4 = V_1 \times \exp(\frac{1}{2})$$

Process 3-4

$$\Delta U = W + Q$$

$$nC_V(T_1 - T_2) = +W + Q$$

$$-\frac{C_V}{R}(P_1V_1) = P_3(V_4 - V_1)\ Q$$

$$Q_{3-4} = \frac{C_V}{R}P_1V_1 - P_1V_1\left(e^{-1} - 2\right)$$

$$Q_{3-4} = P_1V_1\left(\frac{C_V}{R} + 2 - \frac{1}{e^1}\right)$$

Answer

$$e = 1 - \frac{Q_{out}}{Q_{in}}$$

$$e = \frac{W}{Q_{in}} = \frac{2P_1V_1(2e^{-1}) + B}{2P_1V_1\frac{C_V}{R}}$$

$$B = P_1V_1\left[\ln V_1 - \ln\left[e^{\frac{-1}{2}} \times V_1\right]\right]$$

$$e = \frac{e^{\frac{-1}{2}} + \ln V_1 - \ln\left[e^{\frac{-1}{2}}V_1\right]}{\left(\frac{2C_V}{R}\right)}$$

Answer $\left[\frac{e^{\frac{-1}{2}} + \ln V_1 - \ln\left[e^{\frac{-1}{2}}V_1\right]}{\left(\frac{2C_V}{R}\right)}\right]$

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MIPT

(Page 2)

$$m.gR\left(1-\frac{2\sqrt{3}}{3}\right)+2k\frac{Q^2}{R^2}=2m.gR+2k\frac{Q^2}{4R^2}$$

$$m.gR\left(-1-\frac{2\sqrt{3}}{3}\right)=-2k\frac{Q^2}{R^2}+\frac{k}{2}\frac{Q^2}{R^2}$$

$$m.gR\left(-1-\frac{2\sqrt{3}}{3}\right)=-\frac{3}{2}k\frac{Q^2}{R^2}$$

$$-\frac{2}{3}km.gR^3\left(-1-\frac{2\sqrt{3}}{3}\right)=Q^2$$

$$\left[m.gR^3k\left(+\frac{2}{3}+\frac{4\sqrt{3}}{9}\right)\right]^{\frac{1}{2}}=Q$$

~~Answer~~ $Q=\left[km.gR^3\left(\frac{2}{3}+\frac{4\sqrt{3}}{9}\right)\right]^{\frac{1}{2}}$

Aswer: $\left[m.gR^3k\left(\frac{2}{3}+\frac{4\sqrt{3}}{9}\right)\right]^{\frac{1}{2}}=Q$

3) For each ball

$$\sum F_x=m.a_x$$

$$k\frac{Q^2}{d^2}=m.\frac{dv_x}{dt}$$

$$k\frac{Q^2}{x^2}=m\frac{dv}{dx}\frac{dx}{dt}$$

$$k\frac{Q^2}{x^2}=m\frac{dv}{dx}v$$

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MIPT

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$$k\frac{Q^2}{x^2}dx = v\,dv$$

$$kQ^2\int_{2R}^{d}\frac{1}{x^2}dx = \int_0^{v} v\,dv$$

$$kQ^2\left[-\frac{1}{x}\right]_{2R}^{d} = \frac{v^2}{2}$$

$$kQ^2\left(\frac{1}{2R}-\frac{1}{d}\right) = \frac{v^2}{2}$$ the ~~#~~speed reaches its maximum value when $\frac{1}{d}=0$

~~and it gets closer to~~

$$\lim_{d\to\infty}\frac{v^2}{2} = kQ^2\frac{1}{2R}$$

$$\lim_{d\to\infty}\frac{1}{d}=0$$

$$\Rightarrow\frac{u^2}{2} = kQ^2\frac{1}{2R}$$

$$u = \sqrt{k}\,Q\frac{1}{\sqrt{R}}$$

Asver: $u = \sqrt{\frac{k^2}{R}mgR^{3}\left(\frac{2}{3}+\frac{4\sqrt{3}}{9}\right)}$

$$u = kR\sqrt{mg\left(\frac{2}{3}+\frac{4\sqrt{3}}{9}\right)}$$

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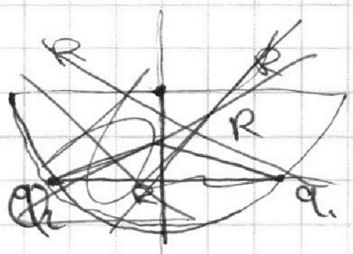
MIPT

5. 1. $\Delta E = 0$

$$E_{p_1} + E_{k_1} + E_{s_1} = E_{p_2} + E_{k_2} + E_{s_2}$$



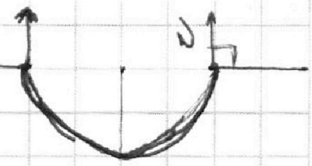
$$h_0 = R - \frac{2R}{\sqrt{3}}$$

$$h_0 = R\left(1 - \frac{2\sqrt{3}}{3}\right)$$

$$mgh_0 + 0 + 2k\frac{Q^2}{R^2} = m \cdot gR + \frac{1}{2}mv^2 + 2k\frac{Q^2}{4(R)^2}$$

(1) $$mgR\left(1 - \frac{2\sqrt{3}}{3}\right) + 2k\frac{Q^2}{R^2} = m \cdot gR + \frac{1}{2}mv^2 + \frac{Q^2}{4(R^2)}2k$$

The velocity vector is perpedicular to the radius of the spheare



$\Rightarrow$ the balls velocities vectors are parallel

$d =$ const for the flight

$\Delta E_k + \Delta E_p + \Delta E_s = 0$ $\qquad \Delta E_s = 0$

$-\frac{1}{2}mv^2 + m \cdot gR = 0$

Aswer: $v^2 = 2gR$ $\qquad v = \sqrt{2gR}$

2] from (1)

$$mgh_0 + 2k\frac{Q^2}{R^2} = m \cdot gR + m \cdot gR + \frac{k}{2}\frac{Q^2}{R^2}$$

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MIPT

isobaric
(1.2) ~~isovolumetric~~ (2-3) isothermal $\frac{C_v}{R}$ $\gamma > 1$

$\Delta U = W + Q$ $\frac{C_p}{C_v} \times C_v$

$C_p - C_v = R$

P = constant $\gamma C_v - C_v = R$

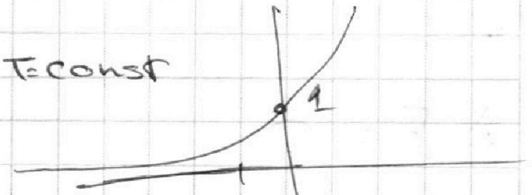
$W_{12} = \int p\,dv$ $C_v(\gamma - 1) = R$

$C_v = \frac{R}{(\gamma - 1)}$

$W = P_0(\Delta V) = P_0(2V_0) = 2P_0V_0$

$PV = nRT$ T=const

~~$P = \frac{nRT}{V}$ $W = \int_{2V_0}$~~



$\frac{-P_1V_1}{2R}$

$\Delta U_{34} = W + Q$ $\Delta U = 0$ for isothermal process

$-W = Q$

$\frac{N}{m^2} \times m^3$

$\Delta U = W + Q$

$W = \int_{V_1}^{V_2} P\,dV$ P = const

$W = P_1(V_2 - V_1) = P_1(2V_1 - V_1) = P_1V_1$

$P_1V_1 = nRT_1$

$P_1 2V_1 = nRT_2$

46 $\Delta U = P_1V_1 + Q$
5 × $nC_v\Delta T = P_1V_1 + Q$
230 $nC_v T_1 = P_1V_1 + Q$
$nC_vT_1 - P_1V_1 = Q_{12}$

$\frac{1}{2} = \frac{T_1}{T_2}$ $T_2 = 2T_1$

230 + 70

113
23
7
16

$\tan\theta =$ cos

$T_1 = \frac{P_1V_1}{nR}$

$\sqrt{\frac{2 \times 160}{5}}$

$2 \times 2 \times 5$

$nC_v\left(\frac{P_1V_1}{nR}\right) - P_1V_1$

$-P_1V_1(\ln V_1 - \ln(V_1 \exp(\frac{1}{2}))) = Q$

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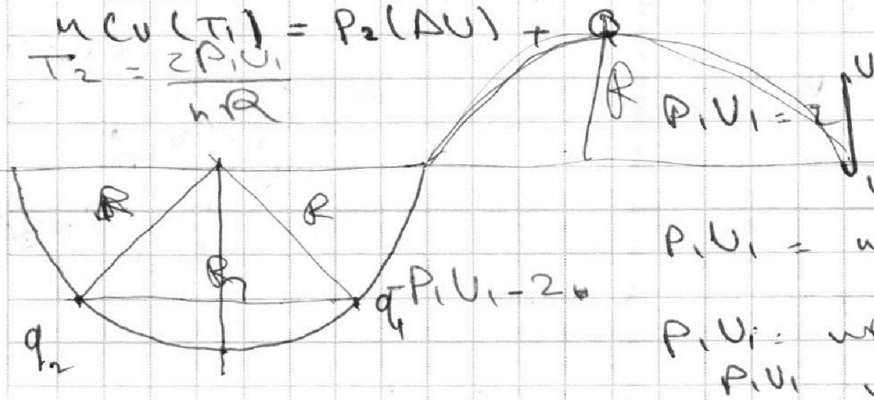
MIPT

$\Delta U = W + Q$

$n C_V (T_1) = P_2(\Delta U) + Q$ — Q is heat added

$T_2 = \frac{2P_1V_1}{nR}$



$P_1V_1 = 2\int_{V_1}^{V_2} \frac{2nRT_2}{V} dV$

$P_1V_1 = nRT\left[\ln(V)\right]_{V_1}^{V_2}$

$P_1V_1 - 2$

$P_1V_1 = nRT(\ln(V_2) - \ln V_1$

$P_1V_1 \quad nRT \quad \ln V_2 - \ln V_1$

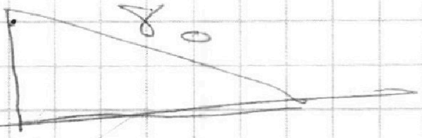
$e^{P_1V_1 - nRT} = e \quad e$

$x V_1 = V_2$

$\Delta E = 0$

~~ΔE_p~~ $E_{p_1} + E_s + E_{K_1} = \Sigma$

$\cos 60 = \frac{\sqrt{3}}{2} = \frac{R}{l}$



$T_2 = \frac{2P_1V_1}{xR}$

$l = \frac{2R}{\sqrt{3}}$

$P_1V_1\left(\frac{2C_V}{Rx} - 1 - \cancel{P}\right)$

$\Delta p = \Sigma_F \cdot \Delta t \quad \frac{1}{n+1} \quad x^{n+1}$

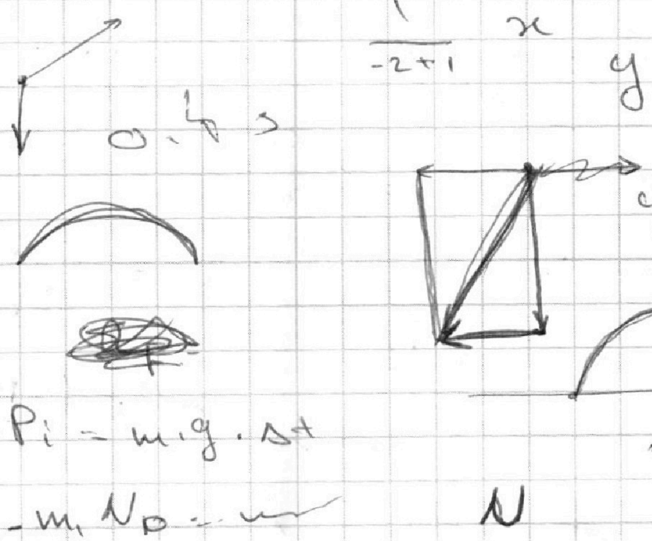
$x = ut$

$t = \frac{x}{u}$

-1

$\frac{1}{-2+1} \quad x^{-1}$

$y = -\frac{1}{2} g t^2 + y_0$



0.45

$v_y = 0$

$y = \frac{1}{2} g \frac{x^2}{u^2}$

$m_1 \times v_x = 30$

$0 = \frac{1}{2} g \frac{80^2}{u^2}$

$P_2 - P_1 = m_1 g \cdot \Delta t$

$t,$

$m_1 v_2 - m_1 v_0 = m$

v

~~$m_1 v_2$~~ $\quad 30 - m_1 u = m_1$

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MIPT

2) in the process 2-3

$$A = \int_{V_2}^{V_3} \frac{2nRT_1}{V} dV \qquad PV = nRT \qquad P = \frac{nRT}{V}$$

$2P_1V_1 +$

$T_2 = 2T_1$

$$-P_1V_1 = 2nRT_1 [\ln(V)]_{V_2}^{V_3} \qquad -A = \int_{V_4}^{V_1} \frac{nRT_1}{V} dV$$

$P_1V_1 = 2nRT_1 \quad \ln V_3 - \ln V_2 \quad P_1V_1 \ln V_1 - \ln$

$$\frac{P_1V_1}{2nRT_1} = \ln V_3 - \ln V_2$$

$\omega = nRT_1(\ln V_1 \ln V_4)$

$\omega =$

$\omega = P_1V_1 (\ln V_1 (\ln(\ln \exp(\frac{1}{2}$

$$e^{\frac{P_1V_1}{} - 2nRT_1} = \frac{V_3}{V_2}$$

$\exp(P_1V_1 - 2nRT_1) \times V_2 = V_3$

$\frac{\omega}{Q_{in}}$

$\exp\left(P_1V_1\left(1 - 2\frac{nR}{nR}\right)\right) \times V_2 = V_3$

$\exp(-P_1V_1)\ V_2 = V_3$

$T_3 = T_2$ (isothermal prosess)

$$P_3 = \frac{2nRT_1}{V_2} \exp(P_1V_1)$$

$T_4 = T_1$ (isothermal prosess)

$$P_3 = \frac{2nR\ P_1V_1}{nR(2\exp(\frac{1}{2})V_1} \qquad P_3 = \frac{P_1}{\exp(\frac{1}{2})V_1}$$

$$\frac{P_1V_1}{\exp(\frac{1}{2})P_1}$$

$P_3 = P_1 \times e^{-0.5}$

$V_1 \times \exp(\frac{1}{2})$

$V_1(\exp(-\frac{1}{2}) - 2\exp(\frac{1}{2})$

$P_1V_1(\exp(-1) - 2)$